WHAT WE DO UNDERSTAND OF COLOUR CONFINEMENT

A. DI GIACOMO

Dipartimento di Fisica dell'Università and INFN, Pisa, Italy[*]

B. LUCINI

Scuola Normale Superiore and INFN, Pisa, Italy[†]

The status of our understanding of confinement is reviewed. The evidence from lattice is that monopole condensation, or dual superconductivity, is at work. Confinement is an order-disorder transition. Different monopole species look equivalent, indicating that the symmetry of the disordered phase is more interesting than we understand.

1 Introduction

No big progress has been made in the last year on the subject. This is a good time to assess what we have learned, and to try an outlook. The question in the title is ambitious. The answer will be: not as much as we think, but we have good handles.

This paper is the sum of two talks: one presented by the first author on the general statement of the problem, the other presented by the second author on specific lattice results. All this is based on results already presented in ref.'s 1. The topics which will be addressed are:

a) lattice vs. continuum formulation of QCD;
b) confinement;
c) duality and disorder parameter;
d) monopoles;
e) monopole condensation, abelian dominance and monopole dominance;
f) what next.

2 Lattice vs. continuum

A popular prejudice is that continuum QCD is based on logical and mathematical arguments, contrary to lattice, which is based on numerical simulations, and therefore does not help in understanding. In reality

[*]e-mail address: digiaco@mailbox.difi.unipi.it
[†]e-mail address: lucini@cibs.sns.it

1. The continuum quantization is perturbative, has the Fock vacuum as ground state, and consists in computing scattering processes between gluons and quarks. Fock vacuum is certainly not the ground state and this instability is signalled by the presence of renormalons, i.e. by the fact that the renormalized perturbative expansion is not even an asymptotic series[2]. However, for some reason, which would be interesting to understand, perturbation theory works at short distances.

2. Lattice formulation is a sensible approximation to the functional Feynman integral which defines the theory. Most probably QCD exists as a self consistent field theory, and is defined constructively on a lattice. Gauge invariance is built in. Since, in addition, objects with non trivial topology (instantons, monopoles) play an important rôle in QCD dynamics, a formulation in terms of parallel transport, like lattice, is superior.

3. Numerical results are like experiments: what is understood from them depends on the question they address. Experiments testing a symmetry, like Michelson-Morley experiment, can be more important to understand than the computation of 3-loop radiative corrections.

3 Confinement

Quarks and gluons have never been observed. The ratio of quark to nucleons abundance in the universe is bounded by the experimental limit

$$\frac{n_q}{n_p} \leq 10^{-27} \tag{1}$$

coming from a Millikan like analysis of $\sim 1g$ of matter. In a cosmological standard model one would expect for the same ratio[3] $n_q/n_p \simeq 10^{-12}$, which is bigger by 15 orders of magnitude.

Lattice numerical evidences exist that QCD confines colour. The Wilson loops obey the area law[4]

$$W(R,T) \underset{RT\to\infty}{\simeq} \exp(-\sigma RT) \ . \tag{2}$$

Since general arguments imply that

$$W(R,T) \underset{RT\to\infty}{\simeq} \exp(-V(R)T) \ , \tag{3}$$

$V(R)$ being the static $Q-\bar{Q}$ potential, it follows that $V(R) = \sigma R$ (σ is the string tension), which means confinement of quarks.

A guiding principle in our analysis of confinement will be that such an absolute property like confinement can only be explained in terms of symmetry.

A similar situation exists e.g. in ordinary superconductivity: the resistivity is observed to be consistent with zero with very great precision. This is not due to the smallness of a tunable parameter, but to a symmetry.

4 Duality

In a finite temperature formulation of QCD the confined phase corresponds to the strong coupling region (low values of $\beta = \frac{2N_c}{g^2}$). Above some β_C deconfinement takes place[5]. Apparently the confined phase is disordered: it should, however, have a nontrivial order, if confinement has to be explained in terms of symmetry.

A wide class of systems exist in statistical mechanics and in field theory, in which a similar situation occurs[6,7]. All those systems have topologically non trivial configurations, carrying a conserved topological charge, and admit two equivalent descriptions (duality). The "ordered" phase (low values of β) is described in terms of the usual local fields, and its symmetry is discussed in terms of vacuum expectation values (*vev*) of local fields (order parameters): in this description topological excitations are extended objects (kinks, vortices, ...). In the dual description the extended topological objects are described in terms of dual local fields, the coupling constant is $\sim 1/g$ and the disordered phase looks ordered, while the phase that originally was ordered looks disordered. The order parameter of the dual description is called a disorder parameter. Understanding confinement consists then in understanding the symmetry of the system dual to QCD. In fact the explicit construction of the dual system is not required: once the dual symmetry is understood the disorder parameter can be constructed in terms of the original local fields, of course as a highly non local operator[7].

A suggestive possibility in this direction is that QCD vacuum could behave as a dual superconductor[8]. Confinement would be a consequence of the dual Meissner effect, squeezing the chromoelectric field of a $Q - \bar{Q}$ pair into an Abrikosov flux tube with energy proportional to the length, or

$$V(R) = \sigma R \ . \tag{4}$$

In this mechanism monopoles are the topological structures which are expected to condense.

Chromoelectric flux tubes are indeed observed on lattice configurations[9] and also their collective modes have been detected[10]. The main results of a more detailed analysis of this mechanism will be presented below.

5 Monopoles in non abelian gauge theories

We shall refer to $SU(2)$ for simplicity: the extension to $SU(3)$ only involves some additional formal complications.

Let $\Phi = \vec{\Phi}\cdot\vec{\sigma}$ be any operator in the adjoint representation. A unit colour vector $\hat{\Phi}(x)$ can be defined:

$$\hat{\Phi}(x) = \frac{\vec{\Phi}(x)}{\mid \vec{\Phi}(x) \mid} \tag{5}$$

everywhere except at sites where $\vec{\Phi}(x) = 0$. The field configuration $\vec{\Phi}(x)$ can present a non trivial topology. If we adopt a "local" reference frame for colour, with 3 orthonormal unit vectors $\vec{\xi}_i(x)$, $\vec{\xi}_i(x)\cdot\vec{\xi}_j(x) = \delta_{ij}$, $\vec{\xi}_i(x)\wedge\vec{\xi}_j(x) = \epsilon_{ijk}\vec{\xi}_k(x)$, with $\vec{\xi}_3(x) = \hat{\Phi}(x)$, instead of the usual x independent unit vectors $\vec{\xi}^o_i$, a rotation $R(x)$ will exist such that

$$\vec{\xi}_i(x) = R(x)\vec{\xi}^o_i \ . \tag{6}$$

Since $\mid \vec{\xi}_i(x) \mid^2 = 1$,

$$\partial_\mu\vec{\xi}_i(x) = \vec{\omega}_\mu \wedge \vec{\xi}_i(x) \tag{7}$$

or

$$D_\mu\vec{\xi}_i(x) = (\partial_\mu - \vec{\omega}_\mu\wedge)\,\vec{\xi}_i(x) = 0 \ . \tag{8}$$

The symbol $\wedge$ indicates cross product; the $SO(3)$ generators are in the fundamental representation $T^a_{ij} = -i\epsilon_{iaj}$.

Eq. (8) implies $[D_\mu, D_\nu] = 0$, or

$$\vec{F}_{\mu\nu} = \partial_\mu\vec{\omega}_\nu - \partial_\nu\vec{\omega}_\mu + \vec{\omega}_\mu \wedge \vec{\omega}_\nu = 0 \ . \tag{9}$$

The rotation (6) is a parallel transport, and as such it is a pure gauge.

The solution of eq. (8) is

$$\vec{\xi}_i(x) = \mathrm{Pexp}\left(i \int_{\infty, C}^{x} \vec{\omega}_\mu \cdot \vec{T}\, dx^\mu \right) \vec{\xi}^o_i \ , \tag{10}$$

and eq. (9) implies that the path integral (10) is independent of the choice of the line C. In fact eq. (9) is not valid at the singularities occurring at the zeros of $\vec{\Phi}(x)$, where $R(x)$ is not defined, and as a consequence $\vec{\xi}_i(x)$ is not independent of the path C.

The inverse rotation $R^{-1}(x)$ acts on $\hat{\Phi}(x) = \vec{\xi}_3$ as

$$R^{-1}(x)\hat{\Phi}(x) = \vec{\xi}^o_3 \ . \tag{11}$$

$R^{-1}(x)$ is called abelian projection.

Under abelian projection the field strength tensor $\vec{G}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + gA_\mu \wedge A_\nu$ has the usual covariant transformation out of the singularities. Where $R(x)$ is not defined it can acquire a singular term

$$\vec{G}_{\mu\nu} \underset{R^{-1}(x)}{\rightarrow} \vec{G}_{\mu\nu} + \hat{\Phi}(x) \left(\partial_\mu A_\nu^{sing} - \partial_\nu A_\mu^{sing}\right) \ . \tag{12}$$

The quantity[11]

$$F_{\mu\nu} = \hat{\Phi}(x) \cdot \vec{G}_{\mu\nu} - \frac{1}{g} \left(D_\mu \hat{\Phi}(x) \wedge D_\nu \hat{\Phi}(x)\right) \cdot \hat{\Phi}(x) \tag{13}$$

can be identically put in the form

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \frac{1}{g} \left(\partial_\mu \hat{\Phi}(x) \wedge \partial_\nu \hat{\Phi}(x)\right) \cdot \hat{\Phi}(x) \ . \tag{14}$$

In the abelian projected form the second term disappears, since $\hat{\Phi}(x) \to \vec{\xi}_3^o$ which is x independent and

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \text{ singular term} \ . \tag{15}$$

$F_{\mu\nu}$ is an abelian field.

$F^\star_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$ defines a magnetic current

$$j^M_\mu = \partial^\nu F^\star_{\mu\nu} \ , \tag{16}$$

which is identically conserved. The system has a magnetic $U(1)$ symmetry. If there are no singularities j^M_μ itself vanishes (Bianchi identities). It can be shown that the singularities of the abelian projection are nothing but pointlike $U(1)$ magnetic charges: the singular term in eq. (12) is a Dirac string taking care of flux conservation.

A magnetic $U(1)$ symmetry exists for each of the functionally-infinite choices of $\vec{\Phi}(x)$: one can construct a disorder parameter as the *vev* of an operator carrying non zero magnetic charge, and investigate dual superconductivity. The disorder parameter can be then measured on the lattice, and the symmetry of the confined vacuum can be determined. This has been done for a number of different choices of the operator $\vec{\Phi}(x)$, and for all of them the transition to confined phase is a transition from dual normal conductor to dual superconductor.

6 Monopole condensation: the disorder parameter

The disorder parameter is the *vev* of a non local operator. The logical procedure to define this operator merely consists in shifting the field variables at a given time t by a configuration describing a static monopole sitting at $\vec{y}$, $\vec{A}(\vec{x},\vec{y})$, in the same way as for a particle moving in one dimension

$$e^{ipa}|x\rangle = |x+a\rangle \ . \tag{17}$$

For a field configuration $\vec{A}(\vec{x},t)$, adding the field $\vec{A}(\vec{x},\vec{y})$ amounts to

$$\mu|\vec{A}(\vec{x},t)\rangle = e^{i\int \vec{\pi}(\vec{x},t)\cdot\vec{A}(\vec{x},\vec{y})\,d\vec{x}}|\vec{A}(\vec{x},t)\rangle = |\vec{A}(\vec{x},t)+\vec{A}(\vec{x},\vec{y})\rangle \ . \tag{18}$$

In doing that care has to be taken of the compactness of the theory, and of the fact that we want to add a monopole to the abelian part of the abelian projected field. All this can be done exactly and gives[12]

$$\langle\mu\rangle = \frac{Z[S+\Delta S]}{Z[S]} \ , \tag{19}$$

where Z is the usual partition function of the theory and ΔS consists in a modification to the action S in the slice $x_0 = t$, in all points of space (non local operator).

The recipe is to change the temporal plaquette $\Pi_{i0}(\vec{n},t)$ to $\Pi'_{i0}(\vec{n},t)$:

$$\Pi_{i0}(\vec{n},t) = U_i(\vec{n},t)U_0(\vec{n}+\hat{\imath},t)\left(U_i(\vec{n},t+1)\right)^\dagger \left(U_0(\vec{n},t)\right)^\dagger \tag{20}$$

$$\Pi'_{i0}(\vec{n},t) = U'_i(\vec{n},t)U_0(\vec{n}+\hat{\imath},t)\left(U_i(\vec{n},t+1)\right)^\dagger \left(U_0(\vec{n},t)\right)^\dagger \ , \tag{21}$$

$$U'_i(\vec{n},t) = e^{-i\Lambda(\vec{n},\vec{y})\hat{\Phi}(\vec{n},t)\cdot\vec{\sigma}}U_i(\vec{n},t)e^{iA^M_{\perp i}(\vec{n}+\hat{\imath}/2,\vec{y})\hat{\Phi}(\vec{n}+\hat{\imath},t)\cdot\vec{\sigma}} \tag{22}$$

$$e^{i\Lambda(\vec{n}+\hat{\imath},\vec{y})\hat{\Phi}(\vec{n}+\hat{\imath},t)\cdot\vec{\sigma}} \ , \tag{23}$$

$\vec{A}^M_\perp(\vec{x},\vec{y})$ and $\Lambda(\vec{x},\vec{y})$ being respectively the transverse ($\vec{\nabla}\cdot\vec{A}^M_\perp(\vec{x},\vec{y})=0$) and the pure gauge part ($\vec{\nabla}\Lambda(\vec{x},\vec{y}) = A_\parallel(\vec{x},\vec{y})$) of $\vec{A}(\vec{x},\vec{y})$.

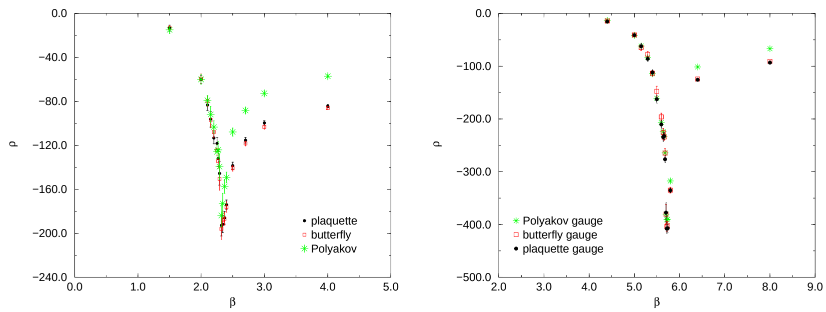
The operator is in fact $\mu = e^{-\beta\Delta S}$, with $\Delta S \sim N_s^3$, N_s being the spatial extension of the system. The fluctuations of $\langle\mu\rangle$ are then $\sim \exp(N_s^{3/2})$. Instead of $\langle\mu\rangle$, which is a widely fluctuating quantity, it proves to be convenient to define

$$\rho = \frac{\mathrm{d}}{\mathrm{d}\beta}\log\langle\mu\rangle \ . \tag{24}$$

Eq. (19) gives

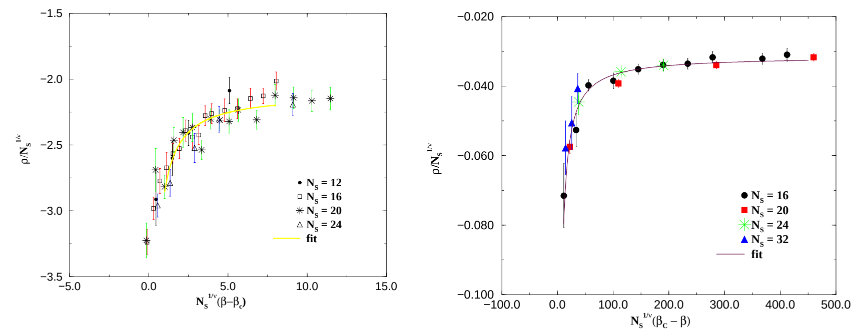
$$\rho = \langle S\rangle_S - \langle S+\Delta S\rangle_{S+\Delta S} \ . \tag{25}$$

The subscript denotes the action used in weighting the average.



(a) (b)

Figure 1. ρ vs. β in different abelian projections for (a) $SU(2)$ and (b) $SU(3)$.



(a) (b)

Figure 2. Rescaled ρ data as a function of the scaling variable for (a) $SU(2)$ and (b) $SU(3)$.

7 Monopole condensation in $SU(2)$ and $SU(3)$

We studied numerically on the lattice the deconfining phase transition at finite temperature for $SU(2)$ and $SU(3)$ by means of the quantity ρ, eq. (25). Finite temperature means that our lattice is $N_s^3 \times N_t$, with $N_s \gg N_t$. N_s^3 is the physical volume, while N_t is related to the temperature T by the relationship $T = 1/\left[N_t a(\beta)\right]$. At finite temperature, $C^\star$ boundary conditions[13] have to be used in the t direction.

We investigated condensation for the monopoles defined by the following operators Φ:

- Φ is related to the Polyakov line $L(\vec{n}, t) = \Pi_{t'=t}^{N_t-1} U_0(\vec{n}, t') \Pi_{t'=0}^{t-1} U_0(\vec{n}, t')$

in the following way:

$$\Phi(n) \equiv \Phi(\vec{n}, t) = L(\vec{n}, t) L^{\star}(\vec{n}, t) \tag{26}$$

(Polyakov projection on a $C^{\star}$ periodic lattice[a]);

- Φ is an open plaquette, i.e. a parallel transport on an elementary square of the lattice

$$\Phi(n) = \Pi_{ij}(\vec{n}, t) = U_i(n) U_j(n+\hat{\imath}) \left(U_i(n+\hat{\jmath})\right)^{\dagger} \left(U_j(n)\right)^{\dagger} \ ; \tag{27}$$

- Φ is the "butterfly" (topological charge density) operator:

$$\Phi(n) = F(\vec{n}, t) = U_x(n) U_y(n+\hat{x}) \left(U_x(n+\hat{y})\right)^{\dagger} \left(U_y(n)\right)^{\dagger} \tag{28}$$
$$U_z(n) U_t(n+\hat{z}) \left(U_z(n+\hat{t})\right)^{\dagger} \left(U_t(n)\right)^{\dagger} \ .$$

Our main results are

1. ρ shows a sharp negative peak in the critical region independently of the abelian projection chosen (fig. 1). $\langle\mu\rangle$ has an abrupt decline in the critical region. ρ does not depend on the abelian projection used.
2. The peak position follows the displacement of the critical temperature when the size in the t direction is changed.
3. In the $SU(3)$ case for a given abelian projection we have two possible choices for defining monopoles, the residual gauge group being $[U(1)]^2$. The corresponding ρ's show a similar behaviour.
4. For both $SU(2)$ and $SU(3)$ at strong coupling (low β's) ρ seems to reach a finite limit when $N_s \to \infty$. From the condition

$$\langle\mu\rangle = \exp\left(\int_0^{\beta} \rho(\beta') \mathrm{d}\beta'\right) \tag{29}$$

it follows that $\langle\mu\rangle \neq 0$ and the magnetic $[U(1)]^{N-1}$ symmetry is broken in the confined phase.
5. At weak coupling (large β's) numerical data are consistent with a linear behaviour of ρ as a function of N_s. For $\langle\mu\rangle$ we get

$$\langle\mu\rangle \underset{N_s \to \infty}{\approx} A e^{(-cN_s + d)\beta} \ , \tag{30}$$

with $c \simeq 0.6$ and $d \simeq -12$ for $SU(2)$ and $c \simeq 2$ and $d \simeq -12$ for $SU(3)$. $\langle\mu\rangle = 0$ in the deconfined phase.

[a] the symbol $\star$ in eq. (26) indicates the complex conjugation operation.

6. A finite size scaling analysis shows that the disorder parameter reproduces the correct critical indices and the correct β_C in both cases. We have indeed

$$\frac{\rho}{N_s^{1/\nu}} = f\left(N_s^{1/\nu}\left(\beta_C - \beta\right)\right) \ , \tag{31}$$

i.e. $\rho/N_s^{1/\nu}$ is a function of the scaling variable $x = \left(N_s^{1/\nu}\left(\beta_C - \beta\right)\right)$. In eq. (31) ν is the critical index associated to the divergence of the correlation length (pseudo-divergence for a first order phase transition) and β_C is the critical value of β. We parameterize $\rho/N_s^{1/\nu}$ as

$$\frac{\rho}{N_s^{1/\nu}} = -\frac{\delta}{x} - c + \frac{a}{N_s^3} \ , \tag{32}$$

where δ is the exponent associated to the drop of $\langle\mu\rangle$ at the transition ($\langle\mu\rangle \propto (\beta_C - \beta)^\delta$, $N_s \to \infty$), c is a constant and a measures scaling violations. We find independently of the abelian projection and (for $SU(3)$) of the abelian generator $\beta_C = 2.29(3)$, $\nu = 0.63(5)$, $\delta = 0.20(8)$, $a \sim 0$ for $SU(2)$ and $\beta_C = 5.69(3)$, $\nu = 0.33(5)$, $\delta = 0.54(4)$, $a \sim 210$ for $SU(3)$ (fig. 2). β_C and ν agree with ref.'s 14.

8 Discussion

The firm point we have is that confinement is an order disorder transition. Whatever the dual symmetry of the disordered phase is, the operators defining dual order have non zero magnetic charge in different abelian projections.

For sure the statement that only one abelian projection is at work, e.g. the maximal abelian, is inconsistent with this symmetry. May be maximal abelian is more convenient than other abelian projections to build effective lagrangeans, but this point is not directly relevant to symmetry.

In addition, if one single abelian projection were involved in monopole condensation, then flux tubes observed on lattice in $Q - \bar{Q}$ configurations should have the electric field belonging to the confining $U(1)$, and this is not the case[15]. Moreover there exist coloured states which are neutral with respect to that $U(1)$, and they would not be confined.

Symmetry of the dual field theory is more clever than we think. It is any how related to condensation of monopoles in different abelian projections. As guessed in ref. 16 all abelian projections are physically equivalent.

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